

Computational Experiments for the Problem of Footstep Planning for Humanoid Robots

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Abstract

It is well known that motion planning problems for humanoid robots are of particular interest. Footstep planning is one of such problems. We describe an approach to solve the footstep planning problem in a free unbounded environment. This approach is based on an explicit reduction from the problem to the satisfiability problem.

Keywords: humanoid robot, footstep planning problem, satisfiability

Many different planning problems are among the most rapidly developing areas of modern robotics. In particular, we can mention different localization problems (see e.g. [1]), path and motion planning problems (see e.g. [2], [3]), visual calibration (see e.g. [4]), the problem of sensor placement (see e.g.

[5] – [8]), allocating complex tasks problems (see e.g. [9] – [12]), automatic generation of visual recognition modules (see e.g. [13] – [15]), the problem of selection of a minimal set of visual landmarks (see e.g. [16] – [18]), selection of partially distinguishable guards (see e.g. [19], [20]), the problem of placement of visual landmarks (see e.g. [21], [22]), systems of robot self-awareness (see e.g. [23] – [27]), the problem of anticipation of motion (see e.g. [28] – [30]). It should be noted that humanoid robotics has recently made rapid progress. However, there is a rising demand for algorithms useful to improving the autonomy of humanoids. In particular, different planning problems for humanoid robots are of considerable interest (see e.g. [31]). Note that motion planning for humanoid robots requires the solution of many difficult problems. For instance, technical vision problems (see e.g. [32] – [34]), problems of selection of visual landmarks, allocating complex tasks problems, footstep planning (see e.g. [35], [36]). Note that the problem of footstep planning is the problem of motion planning associated with walking motion generation. In this case, we need to compute walking motions that bring the robot from its initial location to a goal location while avoiding obstacles. In this paper, we assume that a robot is walking on a flat ground, free from any obstacle. We consider only mean that only discrete stepping capabilities where only a finite set of possible steps is allowed. Under such conditions, the footstep planning problem can be formulated as the reachability problem. In particular, we can consider in the flat infinite ground a Cartesian coordinate system of two axes x and y . We assume that the x axis defines the zero orientation. The configuration of the robot feet in the free environment is completely defined by the position and orientation of the left foot, and the current posture of the feet. So, a sequence of two steps is completely defined by the initial and final posture, and three additional parameters x , y , and θ , where (x, y) is the position and θ is the orientation of the left foot final placement relatively to its initial placement. It is easy to see that we can assume that orientation change is a rational number multiplied by π . Also, we assume that only a finite set of absolute orientations of the left foot are reachable. A posture is defined by the position (x, y) and orientation φ of the right foot relatively to the left foot and by the left foot absolute orientation ψ . Therefore, a posture is a quadruple (u, v, φ, ψ) . Let $\{p[1], \dots, p[n]\}$ is the set of all the reachable postures. Under such conditions, a configuration of the robot is a posture $p[i]$ and a position of the left foot (x, y) . Clearly, we can assume that all the vectors $(p[i], x, y)$ are couples of integer numbers. Similarly, a sequence of two configurations is now completely defined by the initial and final posture and a vector of two parameters. Note that the whole stepping capabilities of the robot can be represented by a 2-counter machine, where $Q = \{p[1], \dots, p[n]\}$ is the set of states. A configuration of the machine is a triple $(p[i], x, y) \in Q \times \mathbb{Z} \times \mathbb{Z}$. It is clear that a configuration of the machine exactly corresponds to a configuration of the robot. Each

allowable sequence of two configurations is a transition $(p[i], x, y, p[j])$. Note that a transition of the machine exactly corresponds to a step of the robot or a change of the robot posture. Reachability Problem (RP): *Given a 2-counter machine with the set of states Q and the set of transitions T and two configurations $(p[i], x, y)$ and $(p[j], u, v)$. Is there a finite sequence of transitions that goes from $(p[i], x, y)$ to $(p[j], u, v)$?* Note that RP is **NP**-complete (see [37]). RP can be considered as the fixed distance problem (see e.g. [37]). Let G be a weighted directed multi-graph with set of nodes $V(G)$ and set of edges $E(G)$. We assume that $W(e) \in Z^k$, $e \in E(G)$, is a cost vector.

THE FIXED DISTANCE PROBLEM (FDP):

INSTANCE: $p, q \in V(G)$, $D \in Z^k$.

QUESTION: *Is there a path e_1, \dots, e_m such that $p = e_1$, $q = e_m$, $\sum W(e_i) = D$?*

Encoding different hard problems as instances of different variants of the satisfiability problem and solving them with very efficient satisfiability algorithms has caused considerable interest (see e.g. [38] – [54]). We consider an explicit reduction from FDP to the satisfiability problem.

We consider the following nondeterministic algorithm (sf. [55]). We add into $E(G)$ a new edge f from q to p such that $W(f) = 0$. We guess a subgraph H of G such that H contains f and forms a strongly connected component. We consider the following ILP instance. There is one variable $c[e]$ for every $e \in E(H)$ where $E(H)$ is the set of edges of H . There are no other variables. The constraints are as follows. Let $c[f] = 1$. Let $c[e] \geq 1$, for all $e \in E(H)$ such that $e \neq f$. Let $V(H)$ be the set of nodes of H . Let $in(r)$ consists of the edges entering r in H . Let $out(r)$ consists of the edges r exiting in H . For all $r \in V(H)$, we assume that $\sum_{e \in in(r)} c[e] = \sum_{e \in out(r)} c[e]$. Let $D = (d_1, d_2, \dots, d_k)$, $W(e) = (w_1(e), w_2(e), \dots, w_k(e))$. For any $1 \leq i \leq k$, we assume that $d_i = \sum_{e \in E(H)} c[e] w_i(e)$. There is a solution to the ILP instance if and only if there is a path e_1, e_2, \dots, e_m from p to q such that $\sum_{i=1}^m W(e_i) = D$ (see [55]). Let $\delta = \max\{\max_{1 \leq i \leq k} \{|d_i|\}, \max_{e \in E(H)} \{\max_{1 \leq i \leq k} \{|w_i(e)|\}\}\}$.

Note that if there is a solution to the ILP instance, then for any $e \in E(H)$ we can assume that $c[e] \leq (|E(G)| + 1)(\delta(k + |V(G)| + 1))^{2(k+|V(G)|+1)+1}$ (sf. [56]). We assume that e exiting $in(e)$ in H and entering $out(e)$ in H . If $d_i \geq 0$, then we assume that $d_i^+ = d_i$, $d_i^- = 0$. If $d_i < 0$, then $d_i^+ = 0$, $d_i^- = |d_i|$. Let

$$\begin{aligned} V(G) &= \{p_1, p_2, \dots, p_{|V(G)|}\}, p_1 = p, p_2 = q, \\ E(G) \cup \{f\} &= \{e_1, e_2, \dots, e_{|E(G)|+1}\}, e_1 = f, \\ M &= \lceil \log_2((|E(G)| + 1)(\delta(k + |V(G)| + 1))^{2(k+|V(G)|+1)+1}) \rceil, \\ c[e_i] &= \sum_{j=0}^M z[i, j] 2^j, d_i^+ = \sum_{j=0}^M d^+[i, j] 2^j, d_i^- = \sum_{j=0}^M d^-[i, j] 2^j, \\ \varphi[1] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |V(G)|} (\neg x[i] \vee \neg x[j] \vee y[1, i, j, 1]), \end{aligned}$$

$$\begin{aligned}
\varphi[2] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |V(G)|} \bigwedge_{1 \leq s \leq |E(G)|} (y[1, i, j, s] \vee \neg y[1, i, j, s+1]), \\
\varphi[3] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq t[1] \leq |E(G)|+1,} (\neg x[i] \vee \neg x[j] \vee \neg y[1, i, j, s] \vee \\
&\quad 1 \leq j \leq |V(G)|, \quad 1 \leq t[2] \leq |E(G)|+1, \\
&\quad 1 \leq s \leq |E(G)| \quad out(e_{t[1]}) \neq in(e_{t[2]})) \\
&\quad \neg y[1, i, j, s+1] \vee \neg y[2, i, j, s, t[1]] \vee \neg y[2, i, j, s+1, t[2]]), \\
\varphi[4] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq t \leq |E(G)|+1,} (\neg x[i] \vee \neg x[j] \vee \neg y[2, i, j, 1, t]), \\
&\quad 1 \leq j \leq |V(G)| \quad p_i \neq in(e_t) \\
\varphi[5] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |V(G)|} \bigwedge_{1 \leq s \leq |E(G)|} \bigwedge_{1 \leq t \leq |E(G)|+1, p_j \neq out(e_t)} (\neg x[i] \vee \\
&\quad \neg x[j] \vee \neg y[1, i, j, s] \vee y[1, i, j, s+1] \vee \neg y[2, i, j, s, t]), \\
\varphi[6] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |V(G)|} \bigwedge_{1 \leq t \leq |E(G)|+1, p_j \neq out(e_t)} (\neg x[i] \vee \\
&\quad \neg x[j] \vee \neg y[1, i, j, |E(G)|+1] \vee \neg y[2, i, j, |E(G)|+1, t]), \\
\varphi[7] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |V(G)|} \bigwedge_{1 \leq s \leq |E(G)|+1} \bigvee_{1 \leq t \leq |E(G)|+1} y[2, i, j, s, t], \\
\varphi[8] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq t[1] \leq |E(G)|+1,} (\neg y[2, i, j, s, t[1]] \vee \neg y[2, i, j, s, t[2]]), \\
&\quad 1 \leq j \leq |V(G)|, \quad 1 \leq t[2] \leq |E(G)|+1, \\
&\quad 1 \leq s \leq |E(G)|+1 \quad t[1] \neq t[2] \\
\varphi[9] &= z[1, 0], \\
\varphi[10] &= \bigwedge_{1 \leq i \leq M} \neg z[1, i], \\
\varphi[11] &= \bigwedge_{2 \leq i \leq |E(G)|+1} \bigvee_{0 \leq j \leq M} z[i, j], \\
\varphi[12] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{0 \leq j \leq M} \neg u[1, i, 0, j], \\
\varphi[13] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1} \neg u[2, i, j, 0], \\
\varphi[14] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in in(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[1, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[2, i, j, s]) \rightarrow \neg u[1, i, j, s], \\
\varphi[15] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in in(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[1, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[2, i, j, s]) \rightarrow \neg u[2, i, j, s+1], \\
\varphi[16] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in in(p_i)} \bigwedge_{0 \leq s \leq M} (u[1, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[2, i, j, s]) \rightarrow u[1, i, j, s], \\
\varphi[17] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in in(p_i)} \bigwedge_{0 \leq s \leq M} (u[1, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[2, i, j, s]) \rightarrow \neg u[2, i, j, s+1], \\
\varphi[18] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in in(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[1, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[2, i, j, s]) \rightarrow u[1, i, j, s], \\
\varphi[19] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in in(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[1, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[2, i, j, s]) \rightarrow \neg u[2, i, j, s+1], \\
\varphi[20] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in in(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[1, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[2, i, j, s]) \rightarrow u[1, i, j, s], \\
\varphi[21] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in in(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[1, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[2, i, j, s]) \rightarrow \neg u[2, i, j, s+1],
\end{aligned}$$

$$\begin{aligned}
\varphi[22] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{in}(p_i)} \bigwedge_{0 \leq s \leq M} (u[1, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[2, i, j, s]) \rightarrow \neg u[1, i, j, s], \\
\varphi[23] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{in}(p_i)} \bigwedge_{0 \leq s \leq M} (u[1, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[2, i, j, s]) \rightarrow u[2, i, j, s+1], \\
\varphi[24] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{in}(p_i)} \bigwedge_{0 \leq s \leq M} (u[1, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[2, i, j, s]) \rightarrow \neg u[1, i, j, s], \\
\varphi[25] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{in}(p_i)} \bigwedge_{0 \leq s \leq M} (u[1, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[2, i, j, s]) \rightarrow u[2, i, j, s+1], \\
\varphi[26] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{in}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[1, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge u[2, i, j, s]) \rightarrow \neg u[1, i, j, s], \\
\varphi[27] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{in}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[1, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge u[2, i, j, s]) \rightarrow u[2, i, j, s+1], \\
\varphi[28] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{in}(p_i)} \bigwedge_{0 \leq s \leq M} (u[1, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge u[2, i, j, s]) \rightarrow u[1, i, j, s], \\
\varphi[29] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{in}(p_i)} \bigwedge_{0 \leq s \leq M} (u[1, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge u[2, i, j, s]) \rightarrow u[2, i, j, s+1], \\
\varphi[30] &= \bigwedge_{1 \leq i \leq |V(G)|, 1 \leq j \leq |E(G)|+1, e_j \notin \text{in}(p_i), 0 \leq s \leq M} u[1, i, j-1, s] = u[1, i, j, s], \\
\varphi[31] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{0 \leq j \leq M} \neg u[3, i, 0, j], \\
\varphi[32] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1} \neg u[4, i, j, 0], \\
\varphi[33] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[3, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[4, i, j, s]) \rightarrow \neg u[3, i, j, s], \\
\varphi[34] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[3, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[4, i, j, s]) \rightarrow \neg u[4, i, j, s+1], \\
\varphi[35] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (u[3, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[4, i, j, s]) \rightarrow u[3, i, j, s], \\
\varphi[36] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (u[3, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[4, i, j, s]) \rightarrow \neg u[4, i, j, s+1], \\
\varphi[37] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[3, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[4, i, j, s]) \rightarrow u[3, i, j, s], \\
\varphi[38] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[3, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[4, i, j, s]) \rightarrow \neg u[4, i, j, s+1], \\
\varphi[39] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[3, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[4, i, j, s]) \rightarrow u[3, i, j, s], \\
\varphi[40] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[3, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[4, i, j, s]) \rightarrow \neg u[4, i, j, s+1],
\end{aligned}$$

$$\begin{aligned}
\varphi[41] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (u[3, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[4, i, j, s]) \rightarrow \neg u[3, i, j, s], \\
\varphi[42] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (u[3, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[4, i, j, s]) \rightarrow u[4, i, j, s+1], \\
\varphi[43] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (u[3, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[4, i, j, s]) \rightarrow \neg u[3, i, j, s], \\
\varphi[44] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (u[3, i, j-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[4, i, j, s]) \rightarrow u[4, i, j, s+1], \\
\varphi[45] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[3, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge u[4, i, j, s]) \rightarrow \neg u[3, i, j, s], \\
\varphi[46] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (\neg u[3, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge u[4, i, j, s]) \rightarrow u[4, i, j, s+1], \\
\varphi[47] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (u[3, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge u[4, i, j, s]) \rightarrow u[3, i, j, s], \\
\varphi[48] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{1 \leq j \leq |E(G)|+1, e_j \in \text{out}(p_i)} \bigwedge_{0 \leq s \leq M} (u[3, i, j-1, s] \wedge \\
&\quad z[j, s] \wedge u[4, i, j, s]) \rightarrow u[4, i, j, s+1], \\
\varphi[49] &= \bigwedge_{1 \leq i \leq |V(G)|, 1 \leq j \leq |E(G)|+1, e_j \notin \text{out}(p_i), 0 \leq s \leq M} u[3, i, j-1, s] = u[3, i, j, s], \\
\varphi[50] &= \bigwedge_{1 \leq i \leq |V(G)|} \bigwedge_{0 \leq s \leq M} \neg x[i] \vee \\
&\quad u[1, i, |E(G)|+1, s] = u[3, i, |E(G)|+1, s], \\
\varphi[51] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1} \bigwedge_{0 \leq s \leq M} \neg u[5, i, j, 0, s], \\
\varphi[52] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1} \bigwedge_{0 \leq t \leq |w_i(e_j)|} \neg u[6, i, j, t, 0], \\
\varphi[53] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1} \bigwedge_{1 \leq t \leq |w_i(e_j)|} \bigwedge_{0 \leq s \leq M} (\neg u[5, i, j, t-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[6, i, j, t, s]) \rightarrow \neg u[5, i, j, t, s], \\
\varphi[54] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1} \bigwedge_{1 \leq t \leq |w_i(e_j)|} \bigwedge_{0 \leq s \leq M} (\neg u[5, i, j, t-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[6, i, j, t, s]) \rightarrow \neg u[6, i, j, t, s+1], \\
\varphi[55] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1} \bigwedge_{1 \leq t \leq |w_i(e_j)|} \bigwedge_{0 \leq s \leq M} (\neg u[5, i, j, t-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[6, i, j, t, s]) \rightarrow u[5, i, j, t, s], \\
\varphi[56] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1} \bigwedge_{1 \leq t \leq |w_i(e_j)|} \bigwedge_{0 \leq s \leq M} (\neg u[5, i, j, t-1, s] \wedge \\
&\quad \neg z[j, s] \wedge u[6, i, j, t, s]) \rightarrow \neg u[6, i, j, t, s+1], \\
\varphi[57] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1} \bigwedge_{1 \leq t \leq |w_i(e_j)|} \bigwedge_{0 \leq s \leq M} (\neg u[5, i, j, t-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[6, i, j, t, s]) \rightarrow u[5, i, j, t, s], \\
\varphi[58] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1} \bigwedge_{1 \leq t \leq |w_i(e_j)|} \bigwedge_{0 \leq s \leq M} (\neg u[5, i, j, t-1, s] \wedge \\
&\quad z[j, s] \wedge \neg u[6, i, j, t, s]) \rightarrow \neg u[6, i, j, t, s+1], \\
\varphi[59] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1} \bigwedge_{1 \leq t \leq |w_i(e_j)|} \bigwedge_{0 \leq s \leq M} (u[5, i, j, t-1, s] \wedge \\
&\quad \neg z[j, s] \wedge \neg u[6, i, j, t, s]) \rightarrow u[5, i, j, t, s],
\end{aligned}$$

[illegible]

$$\begin{aligned}
& \neg u[5, i, j, |w_i(e_j)|, s] \wedge u[8, i, j, s] \rightarrow \neg u[8, i, j, s+1], \\
\varphi[79] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) \geq 0} \wedge_{0 \leq s \leq M} (u[7, i, j-1, s] \wedge \\
& u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[8, i, j, s] \rightarrow \neg u[7, i, j, s], \\
\varphi[80] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) \geq 0} \wedge_{0 \leq s \leq M} (u[7, i, j-1, s] \wedge \\
& u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[8, i, j, s] \rightarrow u[8, i, j, s+1], \\
\varphi[81] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) \geq 0} \wedge_{0 \leq s \leq M} (u[7, i, j-1, s] \wedge \\
& \neg u[5, i, j, |w_i(e_j)|, s] \wedge u[8, i, j, s] \rightarrow \neg u[7, i, j, s], \\
\varphi[82] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) \geq 0} \wedge_{0 \leq s \leq M} (u[7, i, j-1, s] \wedge \\
& \neg u[5, i, j, |w_i(e_j)|, s] \wedge u[8, i, j, s] \rightarrow u[8, i, j, s+1], \\
\varphi[83] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) \geq 0} \wedge_{0 \leq s \leq M} (\neg u[7, i, j-1, s] \wedge \\
& u[5, i, j, |w_i(e_j)|, s] \wedge u[8, i, j, s] \rightarrow \neg u[7, i, j, s], \\
\varphi[84] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) \geq 0} \wedge_{0 \leq s \leq M} (\neg u[7, i, j-1, s] \wedge \\
& u[5, i, j, |w_i(e_j)|, s] \wedge u[8, i, j, s] \rightarrow u[8, i, j, s+1], \\
\varphi[85] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) \geq 0} \wedge_{0 \leq s \leq M} (u[7, i, j-1, s] \wedge \\
& u[5, i, j, |w_i(e_j)|, s] \wedge u[8, i, j, s] \rightarrow u[7, i, j, s], \\
\varphi[86] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) \geq 0} \wedge_{0 \leq s \leq M} (u[7, i, j-1, s] \wedge \\
& u[5, i, j, |w_i(e_j)|, s] \wedge u[8, i, j, s] \rightarrow u[8, i, j, s+1], \\
\varphi[87] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \wedge_{0 \leq s \leq M} u[7, i, j-1, s] = u[7, i, j, s], \\
\varphi[88] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} \neg u[9, i, 0, s], \\
\varphi[89] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1} \neg u[10, i, j, 0], \\
\varphi[90] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \wedge_{0 \leq s \leq M} (\neg u[9, i, j-1, s] \wedge \\
& \neg u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[10, i, j, s] \rightarrow \neg u[9, i, j, s], \\
\varphi[91] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \wedge_{0 \leq s \leq M} (\neg u[9, i, j-1, s] \wedge \\
& \neg u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[10, i, j, s] \rightarrow \neg u[10, i, j, s+1], \\
\varphi[92] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \wedge_{0 \leq s \leq M} (u[9, i, j-1, s] \wedge \\
& \neg u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[10, i, j, s] \rightarrow u[9, i, j, s], \\
\varphi[93] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \wedge_{0 \leq s \leq M} (u[9, i, j-1, s] \wedge \\
& \neg u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[10, i, j, s] \rightarrow \neg u[10, i, j, s+1], \\
\varphi[94] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \wedge_{0 \leq s \leq M} (\neg u[9, i, j-1, s] \wedge \\
& u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[10, i, j, s] \rightarrow u[9, i, j, s], \\
\varphi[95] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \wedge_{0 \leq s \leq M} (\neg u[9, i, j-1, s] \wedge \\
& u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[10, i, j, s] \rightarrow \neg u[10, i, j, s+1], \\
\varphi[96] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \wedge_{0 \leq s \leq M} (\neg u[9, i, j-1, s] \wedge \\
& \neg u[5, i, j, |w_i(e_j)|, s] \wedge u[10, i, j, s] \rightarrow u[9, i, j, s], \\
\varphi[97] &= \wedge_{1 \leq i \leq k} \wedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \wedge_{0 \leq s \leq M} (\neg u[9, i, j-1, s] \wedge
\end{aligned}$$

$$\begin{aligned}
& \neg u[5, i, j, |w_i(e_j)|, s] \wedge u[10, i, j, s] \rightarrow u[10, i, j, s + 1], \\
\varphi[98] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \bigwedge_{0 \leq s \leq M} (u[9, i, j - 1, s] \wedge \\
& \quad u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[10, i, j, s]) \rightarrow \neg u[9, i, j, s], \\
\varphi[99] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \bigwedge_{0 \leq s \leq M} (u[9, i, j - 1, s] \wedge \\
& \quad u[5, i, j, |w_i(e_j)|, s] \wedge \neg u[10, i, j, s]) \rightarrow u[10, i, j, s + 1], \\
\varphi[100] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \bigwedge_{0 \leq s \leq M} (u[9, i, j - 1, s] \wedge \\
& \quad \neg u[5, i, j, |w_i(e_j)|, s] \wedge u[10, i, j, s]) \rightarrow \neg u[9, i, j, s], \\
\varphi[101] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \bigwedge_{0 \leq s \leq M} (u[9, i, j - 1, s] \wedge \\
& \quad \neg u[5, i, j, |w_i(e_j)|, s] \wedge u[10, i, j, s]) \rightarrow u[10, i, j, s + 1], \\
\varphi[102] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \bigwedge_{0 \leq s \leq M} (\neg u[9, i, j - 1, s] \wedge \\
& \quad u[5, i, j, |w_i(e_j)|, s] \wedge u[10, i, j, s]) \rightarrow \neg u[9, i, j, s], \\
\varphi[103] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \bigwedge_{0 \leq s \leq M} (\neg u[9, i, j - 1, s] \wedge \\
& \quad u[5, i, j, |w_i(e_j)|, s] \wedge u[10, i, j, s]) \rightarrow u[10, i, j, s + 1], \\
\varphi[104] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \bigwedge_{0 \leq s \leq M} (u[9, i, j - 1, s] \wedge \\
& \quad u[5, i, j, |w_i(e_j)|, s] \wedge u[10, i, j, s]) \rightarrow u[9, i, j, s], \\
\varphi[105] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) < 0} \bigwedge_{0 \leq s \leq M} (u[9, i, j - 1, s] \wedge \\
& \quad u[5, i, j, |w_i(e_j)|, s] \wedge u[10, i, j, s]) \rightarrow u[10, i, j, s + 1], \\
\varphi[106] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{1 \leq j \leq |E(G)|+1, w_i(e_j) \geq 0} \bigwedge_{0 \leq s \leq M} u[9, i, j - 1, s] = u[9, i, j, s], \\
\varphi[107] &= \bigwedge_{1 \leq i \leq k} \neg u[12, i, 0], \\
\varphi[108] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (\neg u[7, i, |E(G)| + 1, s] \wedge \\
& \quad \neg d^-[i, s] \wedge \neg u[12, i, s]) \rightarrow \neg u[11, i, s], \\
\varphi[109] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (\neg u[7, i, |E(G)| + 1, s] \wedge \\
& \quad \neg d^-[i, s] \wedge \neg u[12, i, s]) \rightarrow \neg u[12, i, s + 1], \\
\varphi[110] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (u[7, i, |E(G)| + 1, s] \wedge \\
& \quad \neg d^-[i, s] \wedge \neg u[12, i, s]) \rightarrow u[11, i, s], \\
\varphi[111] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (u[7, i, |E(G)| + 1, s] \wedge \\
& \quad \neg d^-[i, s] \wedge \neg u[12, i, s]) \rightarrow \neg u[12, i, s + 1], \\
\varphi[112] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (\neg u[7, i, |E(G)| + 1, s] \wedge \\
& \quad d^-[i, s] \wedge \neg u[12, i, s]) \rightarrow u[11, i, s], \\
\varphi[113] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (\neg u[7, i, |E(G)| + 1, s] \wedge \\
& \quad d^-[i, s] \wedge \neg u[12, i, s]) \rightarrow \neg u[12, i, s + 1], \\
\varphi[114] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (\neg u[7, i, |E(G)| + 1, s] \wedge \\
& \quad \neg d^-[i, s] \wedge u[12, i, s]) \rightarrow u[11, i, s], \\
\varphi[115] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (\neg u[7, i, |E(G)| + 1, s] \wedge \\
& \quad \neg d^-[i, s] \wedge u[12, i, s]) \rightarrow \neg u[12, i, s + 1],
\end{aligned}$$

$$\begin{aligned}
\varphi[116] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (u[7, i, |E(G)| + 1, s] \wedge \\
&\quad d^-[i, s] \wedge \neg u[12, i, s]) \rightarrow \neg u[11, i, s], \\
\varphi[117] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (u[7, i, |E(G)| + 1, s] \wedge \\
&\quad d^-[i, s] \wedge \neg u[12, i, s]) \rightarrow u[12, i, s + 1], \\
\varphi[118] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (u[7, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^-[i, s] \wedge u[12, i, s]) \rightarrow \neg u[11, i, s], \\
\varphi[119] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (u[7, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^-[i, s] \wedge u[12, i, s]) \rightarrow u[12, i, s + 1], \\
\varphi[120] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (\neg u[7, i, |E(G)| + 1, s] \wedge \\
&\quad d^-[i, s] \wedge u[12, i, s]) \rightarrow \neg u[11, i, s], \\
\varphi[121] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (\neg u[7, i, |E(G)| + 1, s] \wedge \\
&\quad d^-[i, s] \wedge u[12, i, s]) \rightarrow u[12, i, s + 1], \\
\varphi[122] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (u[7, i, |E(G)| + 1, s] \wedge \\
&\quad d^-[i, s] \wedge u[12, i, s]) \rightarrow u[11, i, s], \\
\varphi[123] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (u[7, i, |E(G)| + 1, s] \wedge \\
&\quad d^-[i, s] \wedge u[12, i, s]) \rightarrow u[12, i, s + 1], \\
\varphi[124] &= \wedge_{1 \leq i \leq k} \neg u[14, i, 0], \\
\varphi[125] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (\neg u[9, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^+[i, s] \wedge \neg u[14, i, s]) \rightarrow \neg u[13, i, s], \\
\varphi[126] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (\neg u[9, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^+[i, s] \wedge \neg u[14, i, s]) \rightarrow \neg u[14, i, s + 1], \\
\varphi[127] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (u[9, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^+[i, s] \wedge \neg u[14, i, s]) \rightarrow u[13, i, s], \\
\varphi[128] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (u[9, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^+[i, s] \wedge \neg u[14, i, s]) \rightarrow \neg u[14, i, s + 1], \\
\varphi[129] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (\neg u[9, i, |E(G)| + 1, s] \wedge \\
&\quad d^+[i, s] \wedge \neg u[14, i, s]) \rightarrow u[13, i, s], \\
\varphi[130] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (\neg u[9, i, |E(G)| + 1, s] \wedge \\
&\quad d^+[i, s] \wedge \neg u[14, i, s]) \rightarrow \neg u[14, i, s + 1], \\
\varphi[131] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (\neg u[9, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^+[i, s] \wedge u[14, i, s]) \rightarrow u[13, i, s], \\
\varphi[132] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (\neg u[9, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^+[i, s] \wedge u[14, i, s]) \rightarrow \neg u[14, i, s + 1], \\
\varphi[133] &= \wedge_{1 \leq i \leq k} \wedge_{0 \leq s \leq M} (u[9, i, |E(G)| + 1, s] \wedge \\
&\quad d^+[i, s] \wedge \neg u[14, i, s]) \rightarrow \neg u[13, i, s],
\end{aligned}$$

$$\begin{aligned}
\varphi[134] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (u[9, i, |E(G)| + 1, s] \wedge \\
&\quad d^+[i, s] \wedge \neg u[14, i, s]) \rightarrow u[14, i, s + 1], \\
\varphi[135] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (u[9, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^+[i, s] \wedge u[14, i, s]) \rightarrow \neg u[13, i, s], \\
\varphi[136] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (u[9, i, |E(G)| + 1, s] \wedge \\
&\quad \neg d^+[i, s] \wedge u[14, i, s]) \rightarrow u[14, i, s + 1], \\
\varphi[137] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (\neg u[9, i, |E(G)| + 1, s] \wedge \\
&\quad d^+[i, s] \wedge u[14, i, s]) \rightarrow \neg u[13, i, s], \\
\varphi[138] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (\neg u[9, i, |E(G)| + 1, s] \wedge \\
&\quad d^+[i, s] \wedge u[14, i, s]) \rightarrow u[14, i, s + 1], \\
\varphi[139] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (u[9, i, |E(G)| + 1, s] \wedge \\
&\quad d^+[i, s] \wedge u[14, i, s]) \rightarrow u[13, i, s], \\
\varphi[140] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} (u[9, i, |E(G)| + 1, s] \wedge \\
&\quad d^+[i, s] \wedge u[14, i, s]) \rightarrow u[14, i, s + 1], \\
\varphi[141] &= \bigwedge_{1 \leq i \leq k} \bigwedge_{0 \leq s \leq M} u[11, i, s] = u[13, i, s].
\end{aligned}$$

Let $\xi = \bigwedge_{i=1}^{141} \varphi[i]$. It is not hard to check that there is a path e_1, \dots, e_m such that $p = e_1$, $q = e_m$, $\sum W(e_i) = D$ if and only if ξ is satisfiable. Using standard transformations we can obtain an explicit transformation ξ into ζ such that $\xi \Leftrightarrow \zeta$ and ζ is a 3-CNF. So, ζ gives us an explicit reduction from FDP to 3SAT. We have designed generators of natural instances for FDP. We consider our genetic algorithms OA[1] (see [22]), OA[2] (see [16]), OA[3] (see [9]), and OA[4] (see [8]) for SAT. We used heterogeneous cluster. Each test was runned on a cluster of at least 100 nodes. Selected experimental results are given in Table 1.

time	OA[1]	OA[2]	OA[3]	OA[4]
average	5.88 h	6.21 h	6.12 h	5.23 h
max	51.37 h	63.38 h	62.46 h	44.32 h
best	11.01 min	42.33 min	46.08 min	14.22 min

Table 1: Experimental results for FDP.

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